MATH 118: Quiz 6

Name: key

Directions:

- Show your thought process (commonly called "showing your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. Suppose $P(x)$ is a polynomial and c is a real number. Write down the definition of a zero for c.
   - c is a x-intercept of the graph of P
   - $x=c$ is a solution to the equation $P(x)=0$. Meaning, $P(c)=0$.
   - $(x-c)$ is a factor of $P(x)$
2. Suppose
$$P(x) = 4x^{122} - 3x^{92} - 5x^{65} + 5x^{19} - 4x^5 + 3x^2 - 1$$
is divided by $x-1$. What is the remainder?

By the remainder theorem, the remainder is $P(1)$.

$$P(1) = 4\cdot 1 - 3\cdot 1 - 5\cdot 1 + 5\cdot 1 - 4\cdot 1 + 3\cdot 1 - 1 = \boxed{-1}$$

3. True or False: A complete factorization of $P(x)$ over $\mathbb{R}$ always results in linear factors.

False. could have irreducibles.

4. Suppose $P(x) = x^3 + 3x^2 + 9x + 27$.

(a) How many zeros does $P(x)$ have?

Degree is 3. So three zeros.

(b) Find a complete factorization over $\mathbb{R}$.

Grouping. $P(x) = x^3 + 3x^2 + 9x + 27$

$= x^2(x+3) + 9(x+3)$

$= \boxed{(x+3)(x^2+9)}$

$x^2 + 9$ irreducible because

$b^2 - 4ac = 0^2 - 4 \cdot 1 \cdot 9$

$= -36$

< 0

(c) Find a complete factorization over $\mathbb{C}$.

Now factor irreducible $x^2 + 9$.

Solve $x^2 + 9 = 0$

$x^2 = -9$

$\sqrt{x^2} = \pm\sqrt{-9}$

$x = \pm i\sqrt{9}$

$x = \pm 3i$

So

$P(x) = (x+3)(x^2+9)$

$= (x+3)(x-3i)(x-(-3i))$

$= \boxed{(x+3)(x-3i)(x+3i)}$